An Introduction to Geometric Measure Theory

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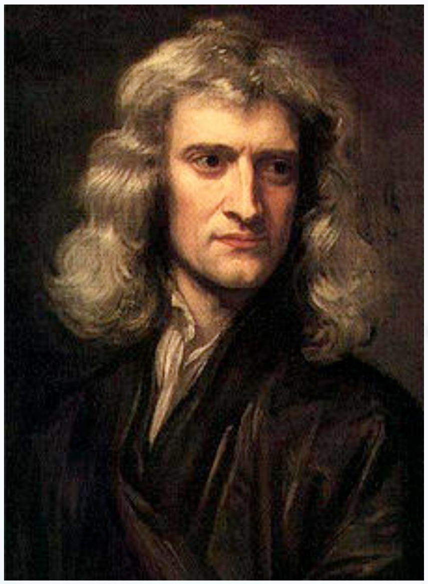
Rice University
31 March, 2011

1684: Calculus

Newton



Hard to overstate his brilliance. Also sort of a jerk.

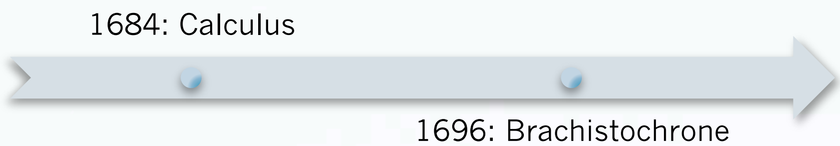
The antiderivative gives area under a curve!

Jinx!

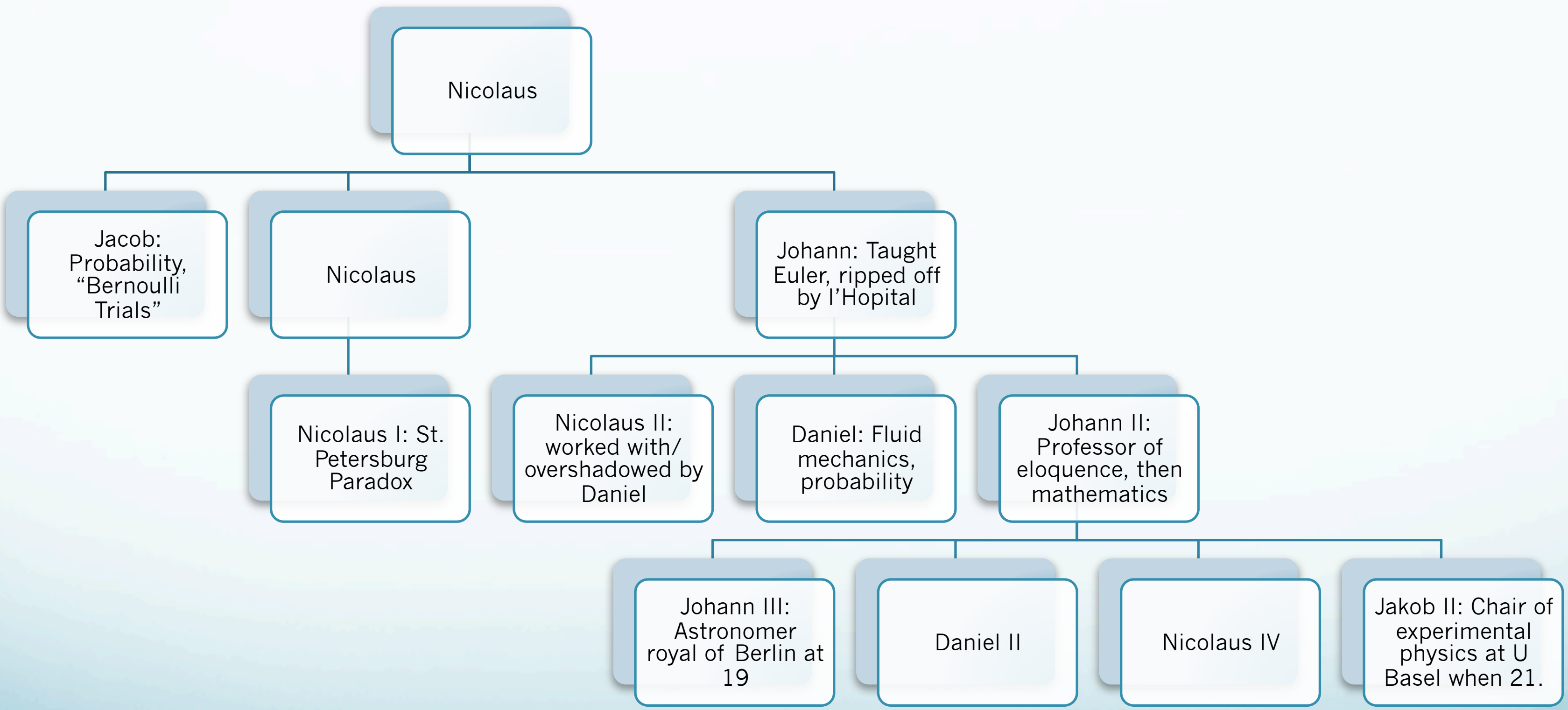
But seriously… I'll ruin your life in a major way if you keep saying that.

Leibniz

Also brilliant. Invented optimism.



Enter the Bernoullis



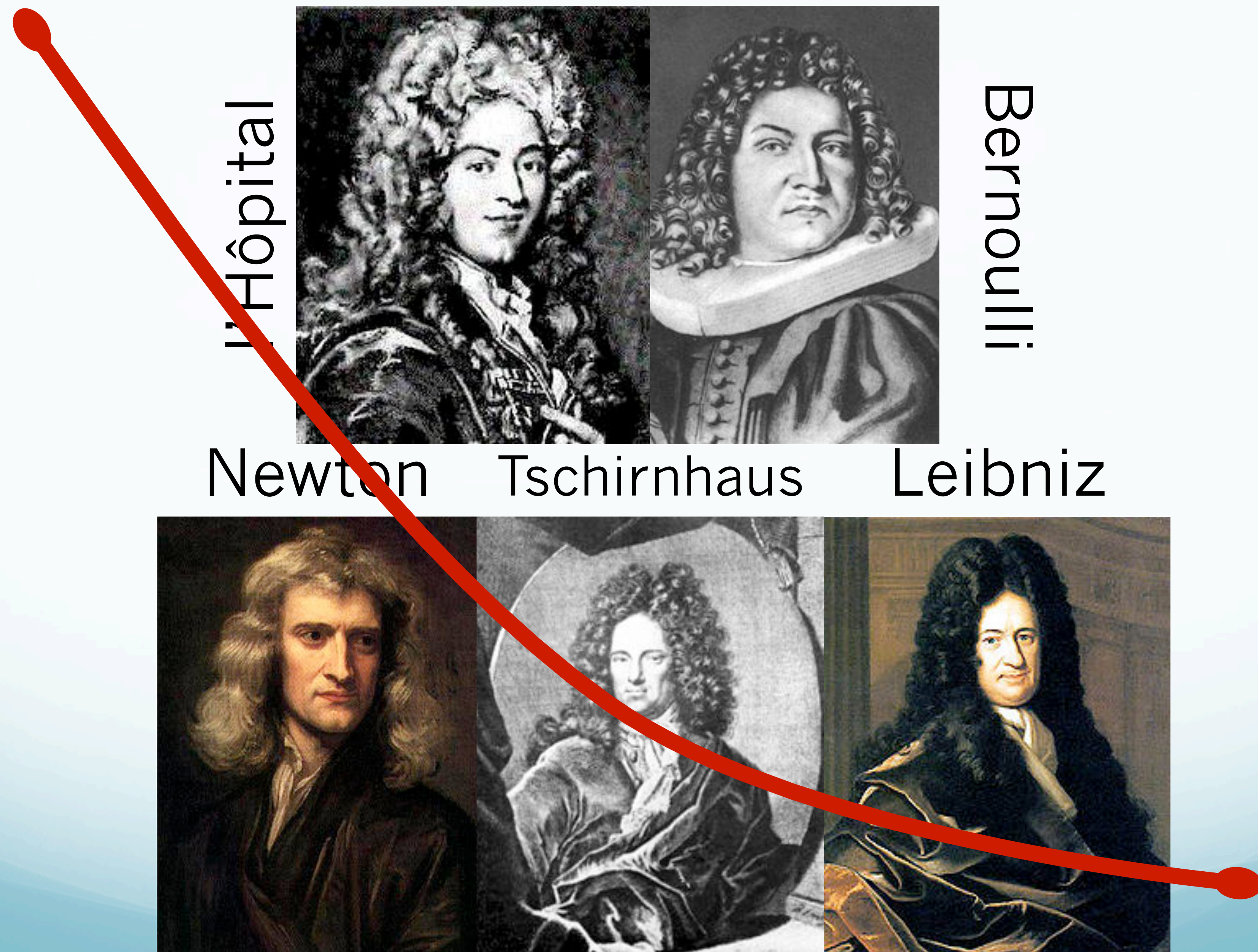
1684: Calculus

1696: Brachistochrone

Enter the Bernoullis

Jacob

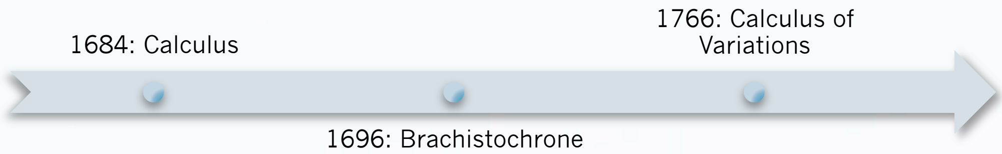
Johann



L'Hôpital
Bernoulli
Newton
Tschirnhaus
Leibniz

I recognize the lion by his paw.

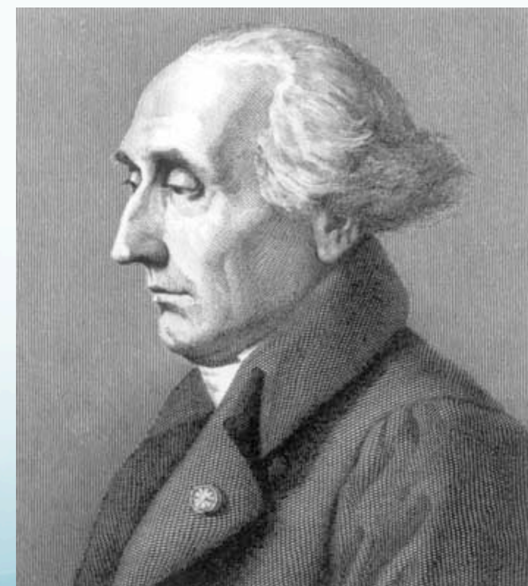
Roar!



The Calculus of Variations

Euler

Lagrange



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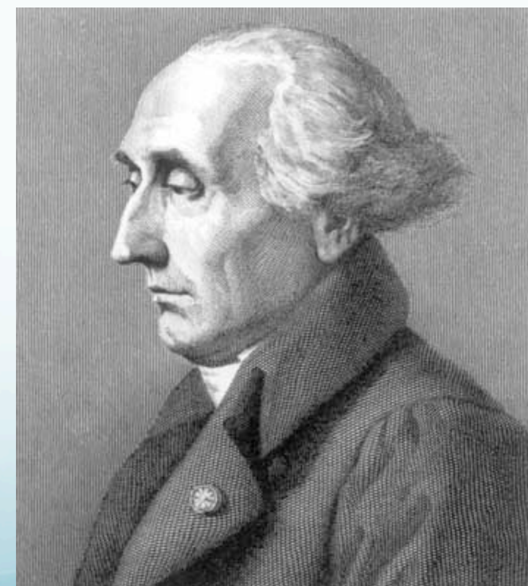
1766: Calculus of Variations

The Calculus of Variations

Find a real valued function *u* that minimizes:

$$F[u] = \int L(u', u, x)\,dx$$

"Functional" "Lagrangian"



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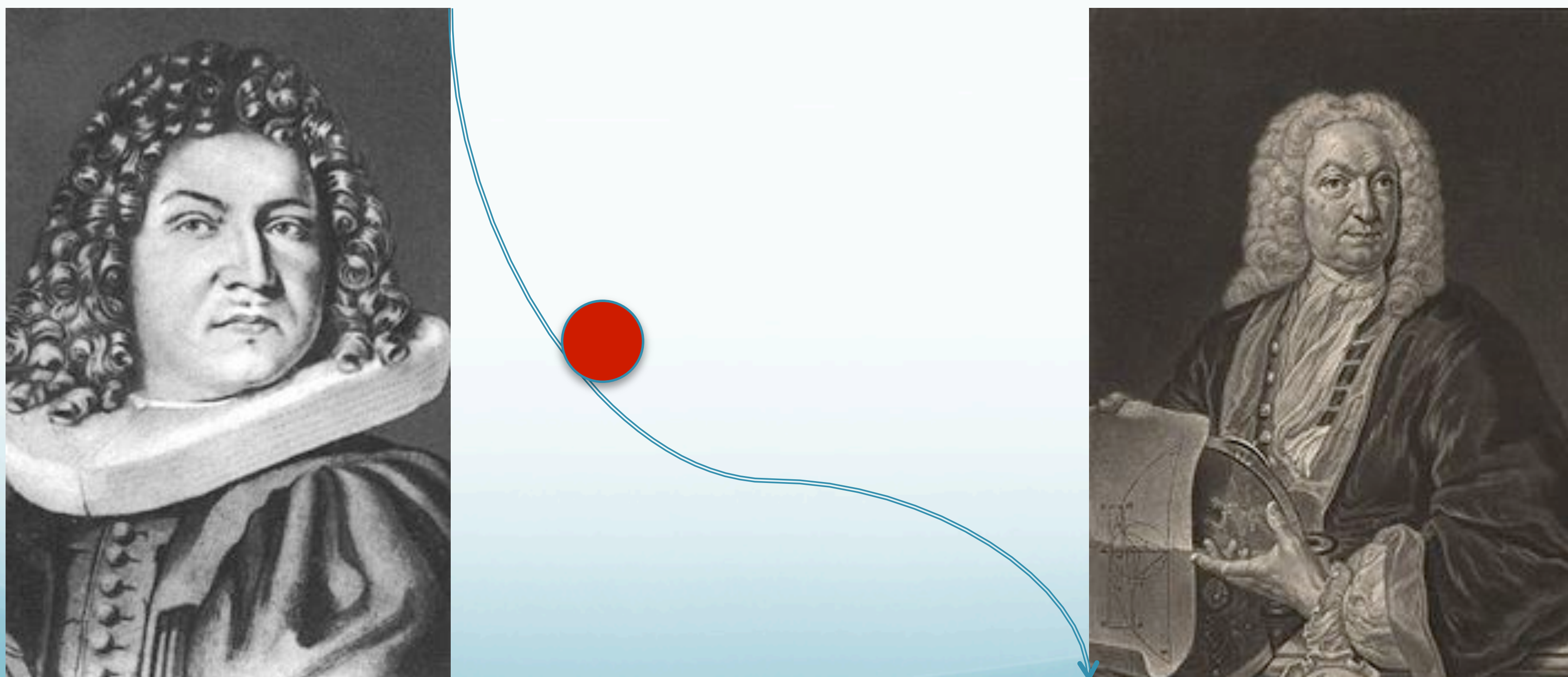
$$F[u] = \int \sqrt{\frac{1 + \left(u'(t)^2\right)}{2g\, u(t)}}\, dt$$

Jacob

Johann



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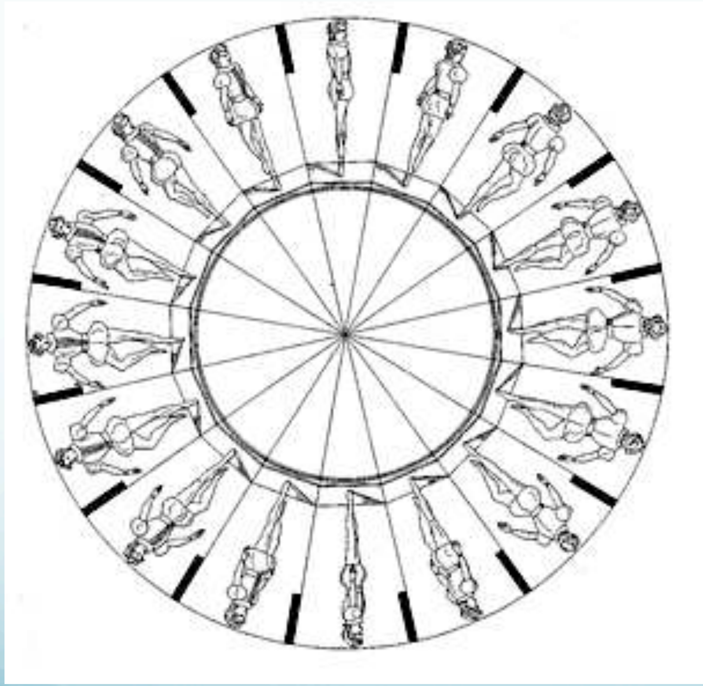
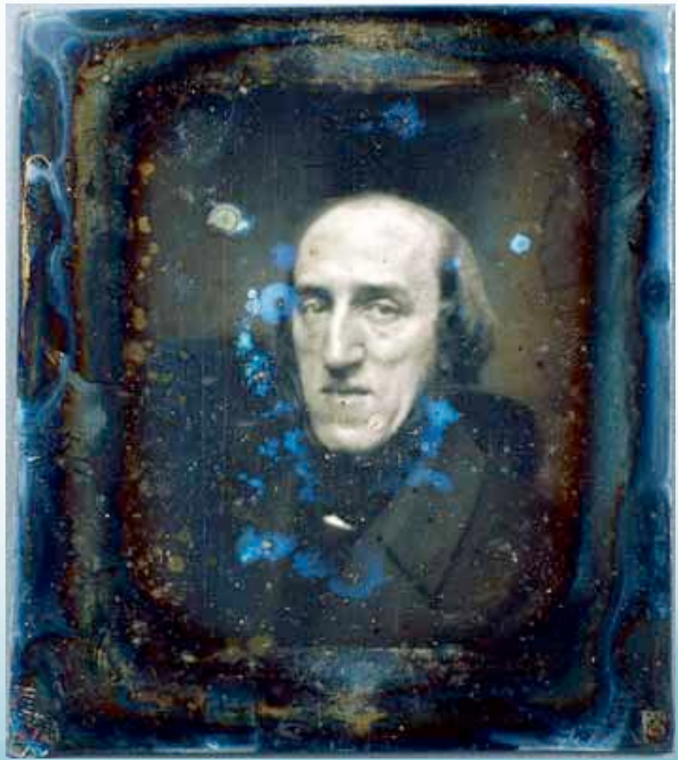
1766: Calculus of Variations

1873: J. Plateau

Joseph Plateau

"Phenakistoscope"





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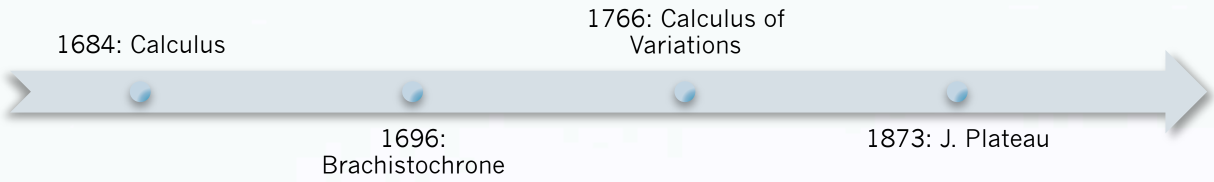
Plateau’s Laws

1. Soap films are made of entire smooth surfaces.
2. The curvature is constant on any piece of soap film.
3. Soap films always meet in threes at 120 degrees (“Plateau border”).
4. Plateau borders meet in fours at a vertex, at a constant angle.



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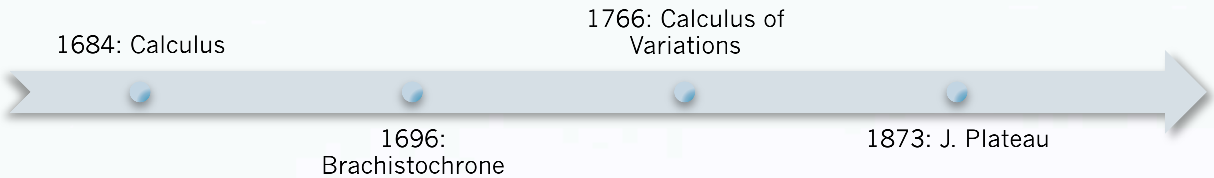
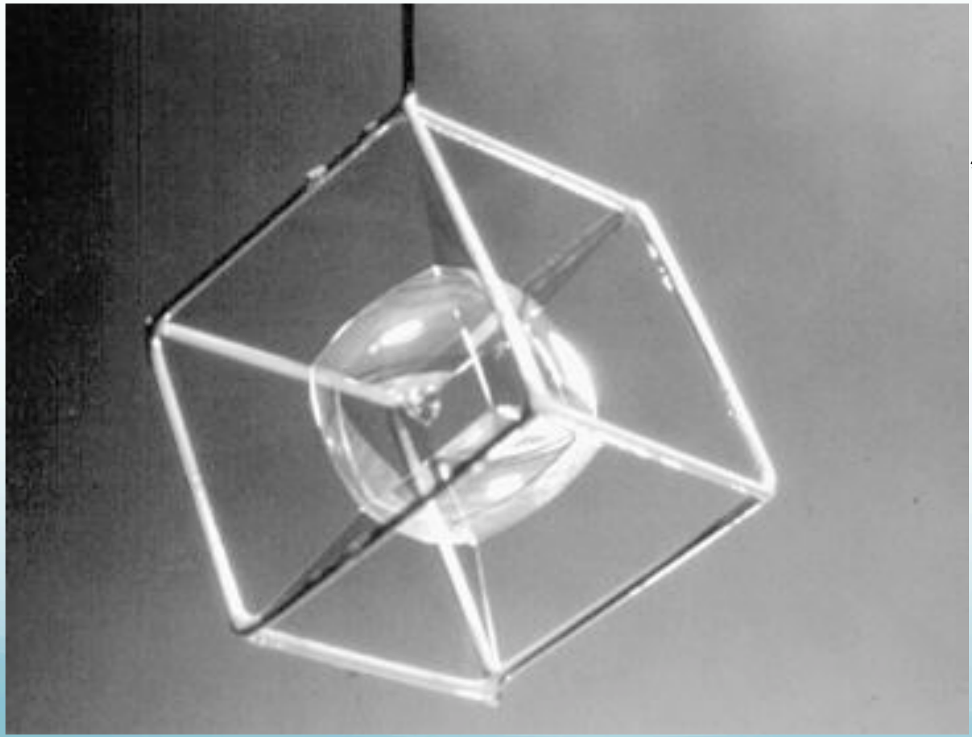
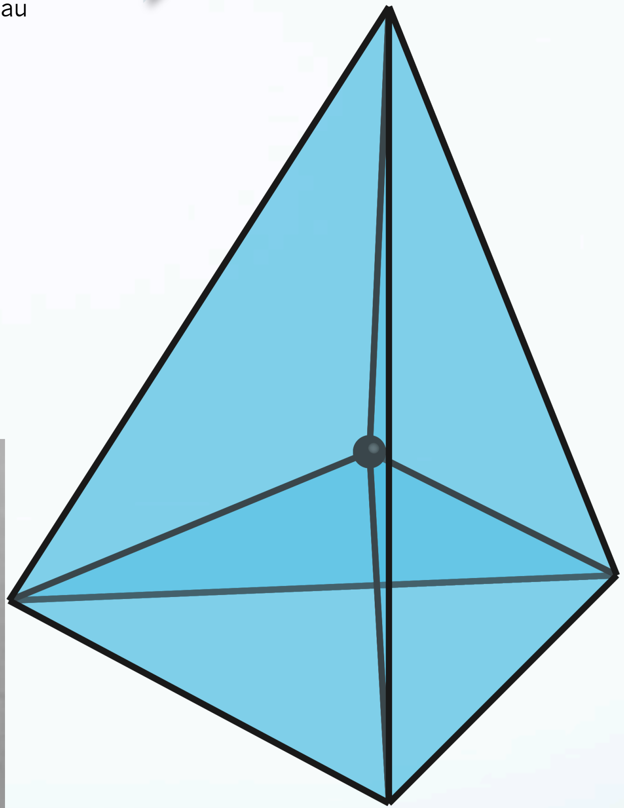


4. Plateau borders meet in fours at a vertex, at a constant angle.





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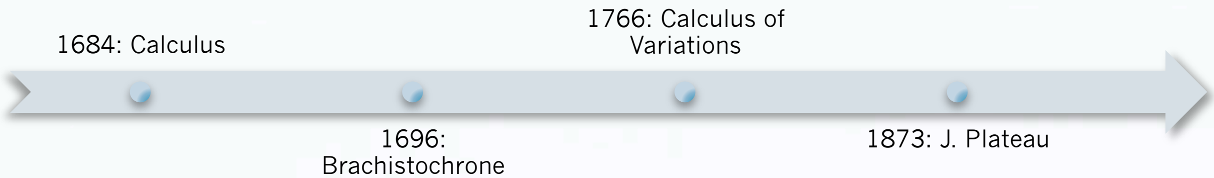
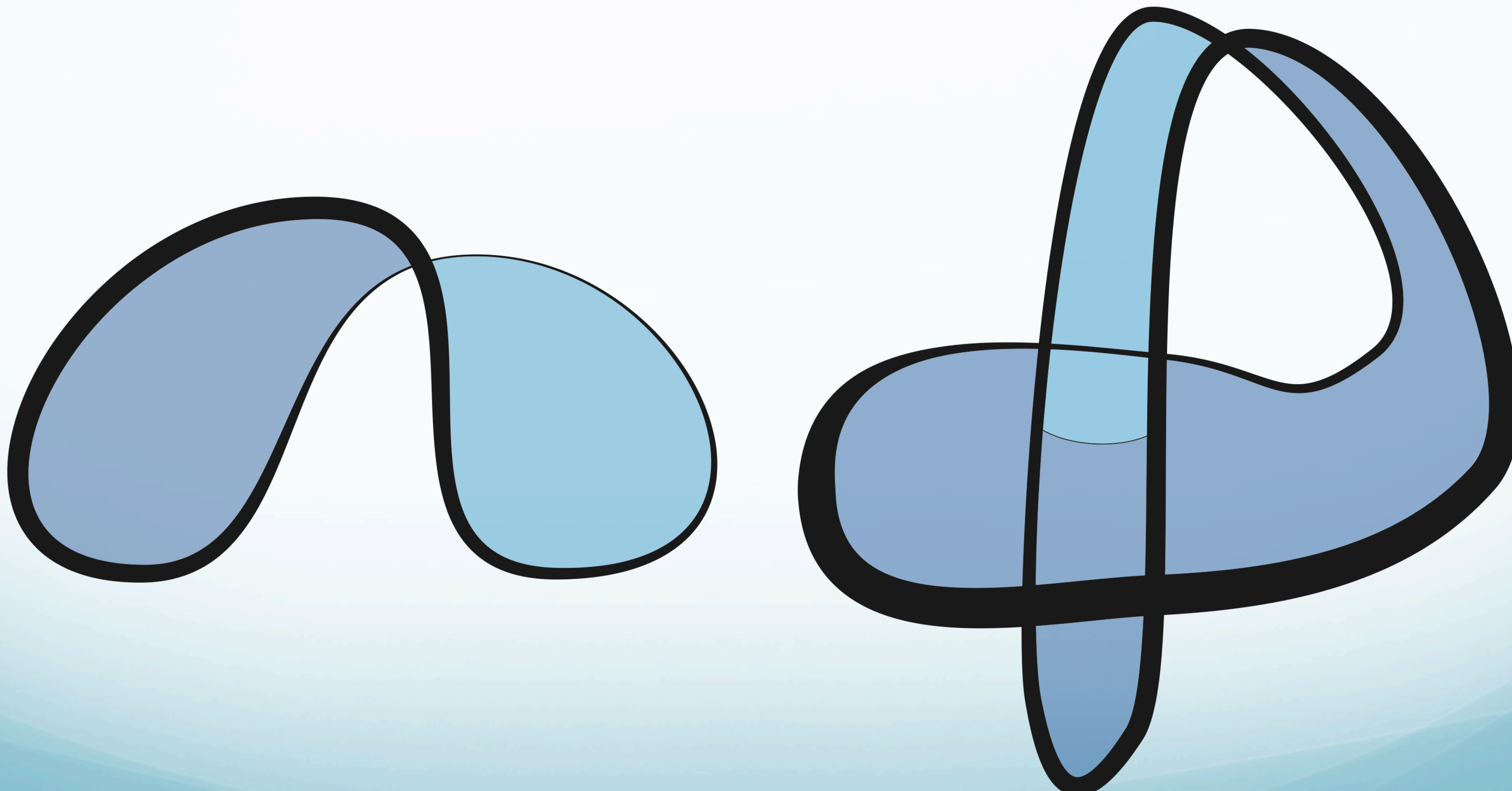
Plateau's Problem

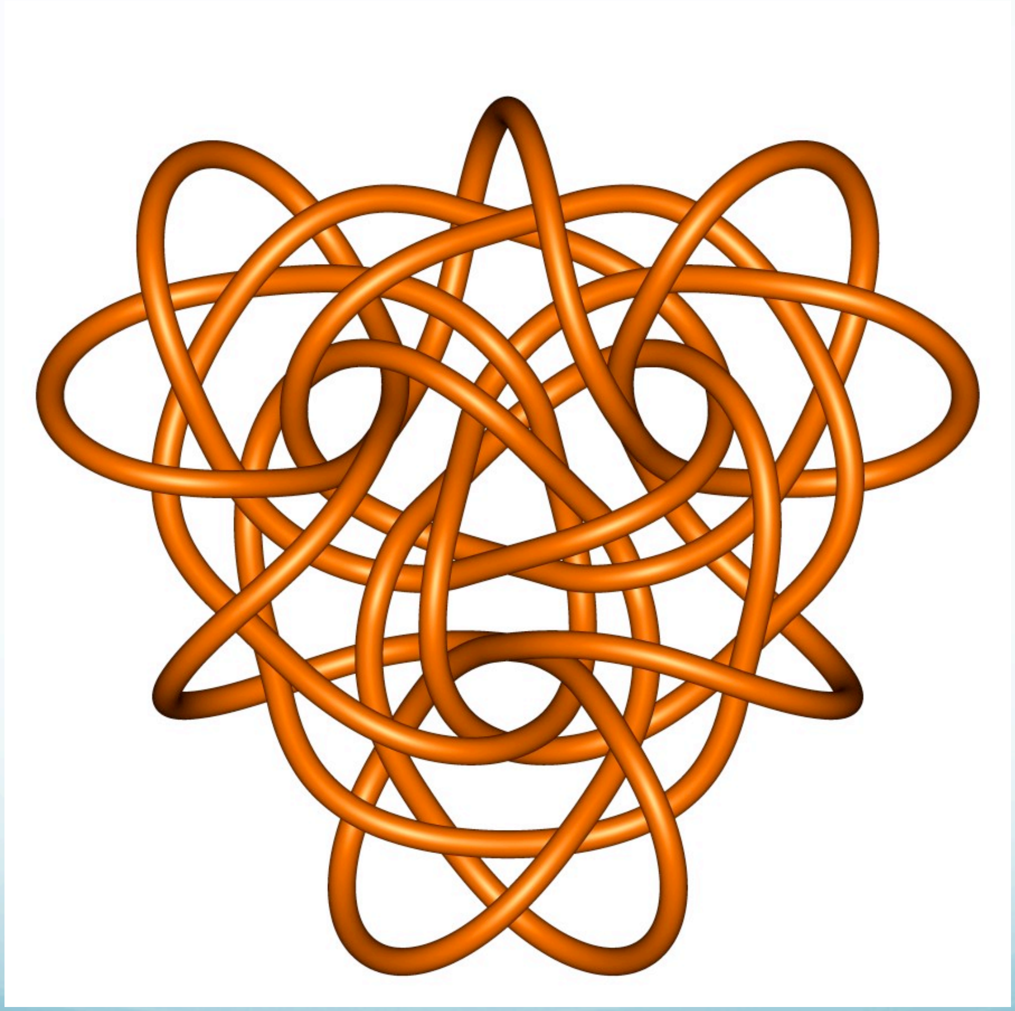
Is there a surface with given boundary that has least area?



Plateau's Problem



Why is this impressive?



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1873: J. Plateau

1930: Solution to Plateau's Problem

Tibor Rado

- Born in Hungary (1895), went briefly to school before enlisting for WWI.
- One of 600,000 taken prisoner on the Russian front.
- Sent to Tobolsk, Siberia. Studied with Helly, an eminent mathematician, in the camp.
- Escaped, befriended eskimos in the arctic, and eventually got back to Hungary.
- Went back to school, studied under Frigyes Riesz.
- Visited Rice Institute in 1929.
- Solved Plateau's problem in 1930.
- Worked at Ohio State until retirement.

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1930: Solution to Plateau's Problem

Jesse Douglas

- Solved Plateau's problem independently and differently.
- Won Field's Medal for his work
- Married a woman named Jessie. Awkward.

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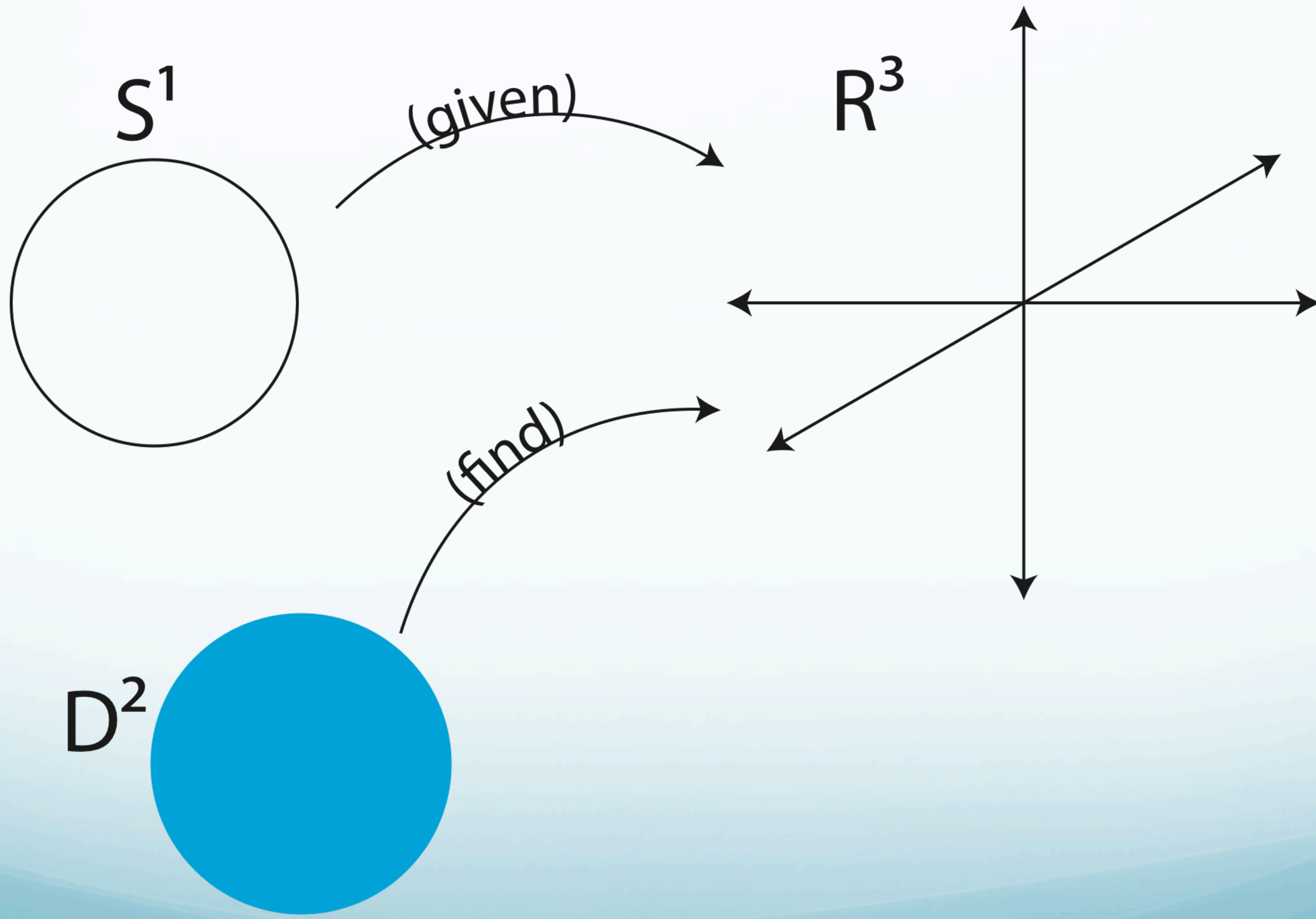
1930: Solution to Plateau's Problem

1960: "Normal and Integral Currents"

Generalizing



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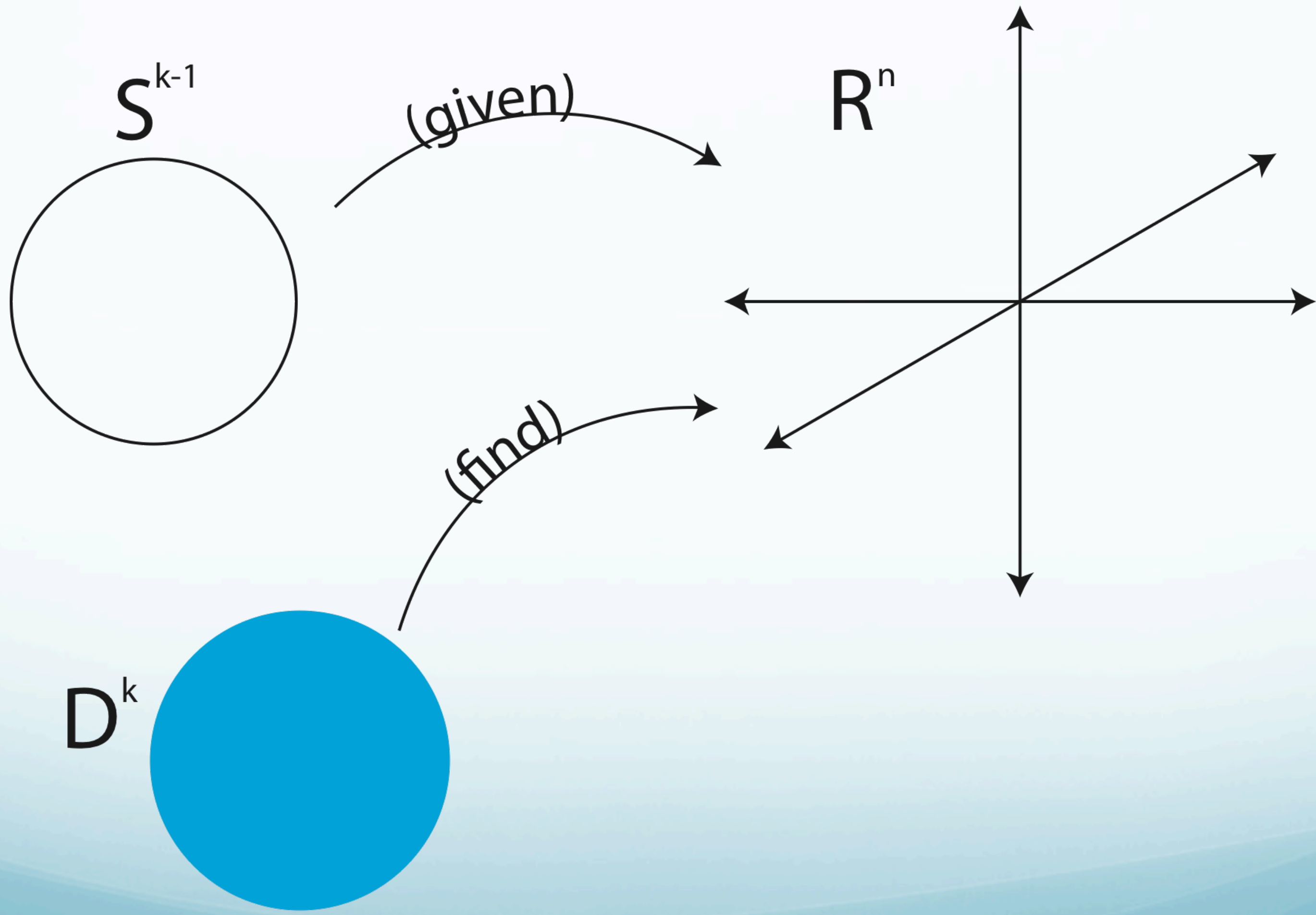

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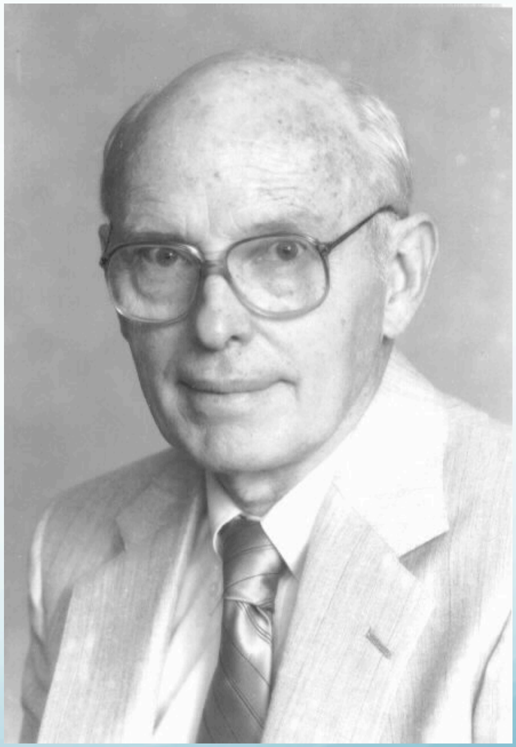
Federer and Fleming



Federer and Fleming

Currents: surfaces::
Complex numbers: real numbers.

$$12x^7 - 40x^6 - 19x^5 + 188x^4 - 157x^3 - 100x^2 + 164x - 48 = 0$$

Next Steps: Regularity

Jean Taylor

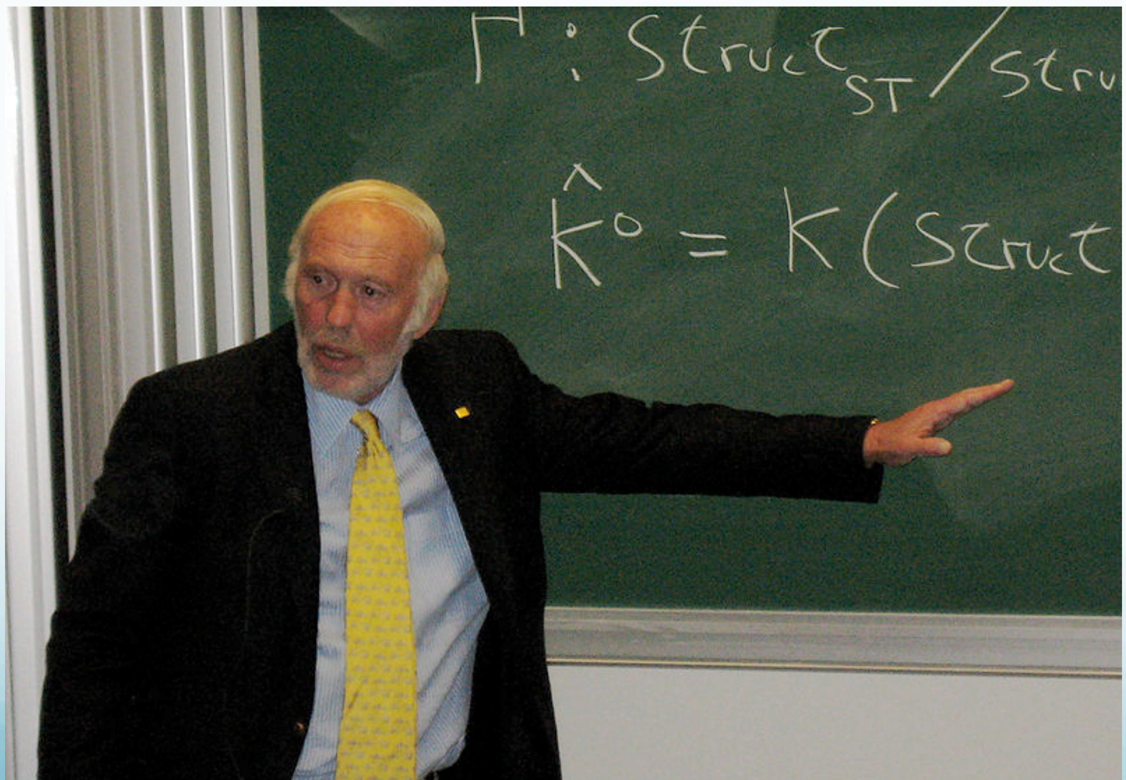
Fred Almgren

How smooth?

Ambient Dimension, n	Dimension of surface	Smoothness of solutions to Plateau's problem
<8	n-1	Real analytic
8 or more	n-1	Might have singularities of dimension n-8.
Any n	Any m<n	Might have singularities of dimension m-2.

Regularity

James Simons. Net worth: $10,600,000,000.00



Isoperimetric Problems

The Double Bubble: 2001, Frank Morgan